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# A simplified model for lateral response of large diameter caisson foundations—Linear elastic formulation

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#### Abstract

The transient response of large embedded foundation elements of length-to-diameter aspect ratio D/B = 2-6 is characterized by a complex stress distribution at the pier–soil interface that cannot be adequately represented by means of existing models for shallow foundations or flexible piles. On the other hand, while three-dimensional (3D) numerical solutions are feasible, they are infrequently employed in practice due to their associated cost and effort. Prompted by the scarcity of simplified models for design in current practice, we here develop an analytical model that accounts for the multitude of soil resistance mechanisms mobilized at their base and circumference, while retaining the advantages of simplified methodologies for the design of non-critical facilities. The characteristics of soil resistance mechanisms and corresponding complex spring functions are developed on the basis of finite element simulations, by equating the stiffness matrix terms and/or overall numerically computed response to the analytical expressions derived by means of the proposed Winkler model. Sensitivity analyses are performed for the optimization of the truncated numerical domain size, the optimal finite element size and the far-field dynamic boundary conditions to avoid spurious wave reflections. Numerical simulations of the transient system response to vertically propagating shear waves are next successfully compared to the analytically predicted response. Finally, the applicability of the method is assessed for soil profiles with depth-varying properties. The formulation of frequency-dependent complex spring functions including material damping is also described, while extension of the methodology to account for nonlinear soil behavior and soil–foundation interface separation is described in the conclusion and is being currently investigated. © 2008 Elsevier Ltd. All rights reserved.

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### 1. Introduction

Drilled shafts or pier foundations are large foundation blocks of intermediate length-to-diameter aspect ratio (typically within the range D/B = 2-6), whose diameter ranges from 2 to 12 ft [1]. Prefabricated and sunk in place, or cast in situ, large diameter caisson foundations are typically used as bridge foundation elements, deep-water wharves, and overpasses. Examples include the Rokko Island Bridge in Kobe, Japan, shown in Fig. 1a, and the Brooklyn Bridge in New York, NY. Small-diameter caissons on the other hand are extensively encountered either as single foundation components of transmission

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towers and heliostats, or in groups as part of the foundation system of high rise buildings, multi-storey parking decks and scour vulnerable structures [1].

Caissons are highly versatile in constructability for a wide variety of soil formations, and can be installed in virtually any soil type including residual soils, karstic formations, soft soils and marine sites. Even further, since no dewatering is necessary during construction, caissons are particularly advantageous at soft sites or sites where excessive groundwater is considered to be critical for the selection of the excavation and support method. The high capacity of single caisson elements in axial as well as lateral loading, and the ability to connect directly to structural members without caps enables them to effectively replace pile groups, and makes them a popular choice for structures anticipated to be subjected to significant lateral

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Fig. 1. (Top) Photo of the foundation system of the Rokko Island Bridge in Kobe, Japan, a double-deck loose arch bridge of length 217 m, constructed in 1992; (middle) schematic illustration of the Rokko Island Bridge geometry indicating the location of caisson foundation elements at the piers adjacent to the bridge abutments (not to scale); (bottom) foundation categorization as a function of the geometry characteristics.

loads. They are thus used worldwide by agencies that focus on the design and construction of lifelines in a wide variety of site conditions, such as the Departments of Transportation.

Categorized according to their geometry characteristics as intermediate embedded foundations (namely of lengthto-diameter ratio 2 < D/B < 6) when compared to shallow embedded footings (D/B < 2) or piles (D/B > 6) (Fig. 1c), caisson foundations are currently designed by means of one of the following two alternative methodologies: (i) existing shallow embedded foundation methods; typical examples include the analytical, semi-analytical, and numerical approaches by Novak and Beredugo [2], Kausel and Roesset [3], Elsabee and Morray [4], Dominguez [5], Tassoulas [6], Mita and Luco [7], Tajirian and Tabatabaie [8], Gazetas [9], and Gazetas and co-workers [10–12], the majority of which have been developed for cylindrical foundations (note that the latter may be applied to foundation elements of arbitrary cross-sections); or (ii) flexible pile approaches (also referred to as p-y and t-z curves) developed semi-empirically as a function of soil type, e.g., Lam and Chaudhury [13]. Alternatively, while three-dimensional (3D) numerical solutions are feasible, their application for the design of non-critical facilities is typically prohibited by the associated site investigation cost, computational time, and user expertise required.

The comparable dimensions of depth to diameter of caisson foundations imply that within the context of assessing the global foundation stiffness, neither the circumference nor the base resistance mechanisms may be neglected. On the other hand, however, pier foundations typically extend through layered soil formations, a fact that requires analytical solutions to be capable of capturing the vertical variability in soil stiffness when simulating the overall stiffness of the soil-foundation system. While the first condition resembles the approach followed within the context of shallow foundation theories, the latter renders p-y curve approaches to be more suitable when accounting for layered media. Results presented in this paper show that caisson foundations are indeed expected to behave as rigid elements similar to shallow foundations for maximum depth to diameter ratios of 6 and typical soil-caisson impedance contrasts, while beyond that range of aspect ratios, their response begins to approach that of flexible piles. Nonetheless, the embedded foundation solutions are shown to be applicable only for low embedment ratios (D/B < 2).

For pier foundations with intermediate length (D/B = 2-6), the soil-structure interaction effects comprising the load-transfer mechanisms from the superstructure to the surrounding soil and the potential altering of loads transferred through the foundation from the soil to the structural elements (e.g., during seismic motion) are associated with a complex stress distribution at the pier-soil interface with comparable contributions from the base and the shaft that cannot be captured by simplified shallow or deep foundation approaches.

Prompted by the scarcity of simplified design methodologies for caisson foundations that may be used to adequately predict their dynamic response at intermediate levels of target design sophistication, we here develop a dynamic Winkler model that properly accounts for the multitude of soil resistance mechanisms mobilized at the base and the circumference of laterally loaded piers—thus retaining the advantages of Winkler-type models while allowing for realistic representation of the complex soil–structure interaction effects associated with these foundation elements.

#### 2. Overview of soil-structure interaction methodologies

The fundamental objective of soil-structure interaction analysis is illustrated in Fig. 2a (modified from Ref. [14]). The formulation of the problem comprises a structure with finite dimensions embedded in soft soil that extends to infinity and specified time-varying loads acting on the structure, originating either from forced vibrations of the superstructure (e.g., rotating machinery) or introduced through the soil by means of incident seismic waves at the foundation level. Objective of the problem is to determine the dynamic response of the structure interacting with the soil.

The various aspects of the problems are qualitatively described in the ensuing, along with the corresponding families of methodologies implemented for the quantification of the associated effects. In particular, when the bounded structure, and any adjacent irregular soil region



Fig. 2. Geometry and decomposition of a soil-structure interaction problem: (a) soil-foundation-structure interaction system, (b) kinematic interaction, and (c) inertial interaction (modified from Ref. [14]).

that can be regarded as part of the structure, are expected to exhibit nonlinear behavior, well-established methods of structural dynamics may be implemented to determine a finite-element model with a finite number of degrees of freedom for the structure. The corresponding nonlinear dynamic equations of motion of the discretized structure can be then formulated and solved directly in the timedomain by means of existing numerical methodologies. Evaluation of the problem solution by means of the aforementioned methodology is referred to as the *direct approach*, which while allowing for the simultaneous mathematical representation of the structure and underlying soil as well as potential nonlinear behavior of either components, remains quite expensive from a computational standpoint, and is thus for the most part implemented in engineering practice for the design of critical structures.

An alternative and computationally efficient approach is the so-called *substructure approach*, according to which the soil-structure interaction problem is decomposed into the three distinct components, namely the soil, the foundation, and the superstructure, which are successively combined to formulate the complete solution. Whitman [15] introduced the terms *inertial* and *kinematic* interaction to describe the aforementioned effects, while studies by Elsabee and Morray [4] confirmed the importance of kinematic interaction effects, particularly for embedded foundations.

The term kinematic interaction refers to the effects of the incident seismic waves to the system shown in Fig. 2b, comprising the foundation and the supporting soil, which differ from the complete system of Fig. 2a since the mass of the superstructure is set equal to zero. On the other hand, *inertial interaction* refers to the response of the complete structure-foundation-soil system to excitation by D' Alembert forces associated with the acceleration of the superstructure due to kinematic interaction (Fig. 2c). It should noted that while the superposition principle is exact only for linear soil, foundation and structure behavior, approximations of soil nonlinearity by means of iterative viscoelastic wave propagation analyses allow superposition to be approximately employed for moderately nonlinear systems. The principal advantage of the substructure approach is the associated numerical flexibility that comprises the following analysis steps [16]: (i) the seismic response of the system in Fig. 2a to incident seismic motion is initially evaluated, while the total relative displacement field is decomposed into its kinematic and inertial components; (ii) successively, the inertial interaction analysis is conducted by computing the foundation dynamic impedance (i.e., springs and dashpots) associated with each mode of vibration, namely the oscillation pattern imposed by the external load (swaying, rocking, etc.), and determining the seismic response of structure and foundation supported by these springs and dashpots and subjected to the kinematic motion of the base.

For each one of the aforementioned analysis steps, several alternative formulations have been developed and published in the literature, including finite-element, boundary-element, semi-analytical and analytical solutions, a variety of simplified methods, and semi-empirical methods. Perhaps the most popular approaches used in practice for the analysis of soil–structure interaction problems are referred to as *Winkler models*. Employed primarily for inertial interaction analyses, the foundation in these methods is supported by a series of independent vertical, rotational and horizontal springs and dashpots along the soil–footing interface, which correspond to the vibration modes. For elastic analyses, the most important factors affecting the dynamic impedance of foundations are: (i) the stiffness modulus of the soil; (ii) the stiffness modulus of the foundation material (e.g., concrete); (iii) the shape of the foundation (e.g., circular vs. square caisson crosssection); (iv) the stratigraphy of the soil profile (common idealizations include homogeneous halfspace, surface soil layer overlying rigid bedrock or elastic halfspace); and (v) the amount of embedment. For the estimation of the dynamic impedance of footings, algebraic expressions have been developed that account for arbitrary foundation shape and degree of embedment, and for a variety of soil conditions. For more details, the reader is referred to Dobry and Gazetas [17], Wong and Luco [18], Gazetas [9], Kausel and Roesset [3], and Luco [19].

In these studies, the dynamic impedance of foundations is shown to be very sensitive to the underlying soil stratigraphy. The response of a foundation on a nonhomogeneous halfspace can be substantially different from the response of an identical foundation resting on a homogeneous halfspace. This effect arises both from the increase of static stiffness and the decrease of radiation damping and is more prominent for the vertical and horizontal oscillations. Subsequently, the amplitude of the motion to be exerted by the supported structure increases as a result of the resonant peaks which appear in the amplitude-frequency response curves (see Refs. [9,20,21]). In the ensuing, a Winkler model is developed and implemented in this study for the dynamic and kinematic interaction analysis of large diameter embedded foundations.

### 3. Winkler model for the analysis of large diameter embedded foundations

Fig. 3 schematically depicts the stress distribution and associated stress resultants developed at the foundationsoil interface, when a typical caisson is subjected to transverse loading at the top, the former here represented by a combination of a lateral concentrated load (V) and a moment (M). As can be readily seen, four mechanisms are identified as significantly contributing to the pier response [22]. The mathematical expressions for the resistance mobilized by these mechanisms are presented below and comprise the following:

(a) Lateral resistance per unit length due to normal stresses along the shaft:

$$P_h = \int_0^{2\pi} [\sigma_r \cos \psi + \tau_{r\psi} \sin \psi] r \,\mathrm{d}\psi \tag{1}$$

(b) Resisting moment per unit length due to vertical shear stress along the shaft:

$$M_h = \int_0^{2\pi} \tau_{rz} \left(\frac{B}{2}\right)^2 \cos \psi \, \mathrm{d}\psi \tag{2}$$



Fig. 3. Primary resistance mechanisms contributing to the overall stiffness of caisson foundations subjected to transverse loading at ground surface: (a) stress distribution and (b) normal and shear stress resultants applied on the circumference and the base of the caisson (modified from Ref. [22]).

(c) Lateral base resistance due to horizontal shear stress:

$$P_b = \int_0^{B/2} \int_0^{2\pi} (-\tau_{rz} \cos \psi + \tau_{\psi z} \sin \psi) r \, \mathrm{d}\psi \, \mathrm{d}r \qquad (3)$$

(d) Base resisting moment due to normal stresses:

$$M_b = \int_0^{B/2} \int_0^{2\pi} (\sigma_z \, \cos \, \psi) r^2 \, \mathrm{d}\psi \, \mathrm{d}r \tag{4}$$

Following Mayne et al. [22], Assimaki et al. [23] and Gerolymos and Gazetas [10–12], a four-spring model is here implemented to capture macroscopically the aforementioned resistance mechanisms (Fig. 4). The distributed and concentrated spring functions, calibrated in the



Fig. 4. Four-spring model implemented in this study for the macroscopic representation of the complex resistance mechanisms developed at the soil–caisson interface (modified from Ref. [23]).

ensuing by means of 3D finite element simulations, comprise the following: (i) lateral translational springs representing the lateral force-displacement soil response  $(k_x)$ ; (ii) rotational springs representing the moment developed at the centerline of pier due to vertical shear stress acting at the perimeter of pier, induced by pier rotation  $(k_{\theta})$ ; (iii) base translational concentrated spring representing the horizontal shear force-base displacement response  $(K_{bx})$ ; and (iv) base rotational spring representing the moment due to normal stress acting at the base of pier, induced by base rotation  $(K_{b\theta})$ .

The model is based on the assumption that the response of each soil layer is uncoupled from the overlying and underlying ones, an approximation of plane strain response of the foundation element originally proposed by Novak et al. [24]. As a result, in absence of coupling between adjacent soil resistance mechanisms, the total response can be obtained through integration of the total resistance offered by the individual springs for each layer. While this assumption is not valid in the immediate vicinity of soil layer interfaces, for soil layers where the cross-layer interaction region represents a small percentage of the total layer thickness (i.e., adequately thick layers), the coupling effect diminishes very rapidly with distance from the interface and the overall contribution of the coupling to total response becomes practically negligible.

In the ensuing, the response of the model shown in Fig. 4 subjected to transverse loading is investigated by means of the Euler beam theory formulation, and simplified deformation profiles are developed for typical geometry characteristics and foundation-soil impedance contrasts. The overall foundation stiffness matrix 'as interpreted from the top of the caisson' by integration of the individual resistance mechanisms is successively evaluated based on the deformation simplifications.

# 4. Approximate caisson deformation based on Euler beam theory

Considering that in the most general case, the response of an embedded foundation element (e.g., the large diameter caisson shown in Fig. 4) subjected to transverse loading at the top can be approximated by means of the Euler beam theory, the governing equation for static deformation of a caisson may be formulated as follows:

$$E_p I_p u'' = M^* \tag{5}$$

where M and V are the concentrated moment and lateral force at the top of the foundation, u is the lateral deformation of the foundation element varying with depth from ground surface, z is the depth from ground surface, and  $E_p$  and  $I_p$  are the foundation Young's modulus and area moment of inertia correspondingly. Differentiating Eq. (5) with respect to z yields the following alternative formulation for the governing equation:

$$(E_p I_p u'')'' = \frac{\partial^2 M^*}{\partial z^2} \tag{6}$$

For the foundation element under investigation, namely a uniform solid cylinder supported by distributed translational  $(k_x)$  and rotational  $(k_\theta)$  springs along the shaft, and a concentrated translational  $(K_{bx})$  and a rotational  $(K_{b\theta})$  spring at the base, the moment resultant on a cross-section at depth z from the surface (Fig. 5) is:

$$M^{*} = M + Vz - \int_{0}^{z} k_{\theta} u^{I}(\tau) \,\mathrm{d}\tau - \int_{0}^{z} k_{x} u(\tau)(z-\tau) \,\mathrm{d}\tau \qquad (7)$$

It should be noted that since the linear elastic idealization of the soil response is valid only in the small deformation range, the equation above is based on the undeformed



Fig. 5. Moment resultant at depth z from ground surface, implemented in the Euler beam formulation for the estimation of the idealized deformed configuration of the foundation element on Winkler soil subjected to lateral loading at the top.

configuration of the soil-foundation system, and as a result second-order geometric effects such as  $p-\delta$  effects have been neglected.

When substituted into Eq. (6), the above Eq. (7) results in the following fourth-order differential formulation describing the deformation of the foundation element, u(z):

$$E_p I_p u^{IV} + k_\theta u'' + k_x u = 0 (8)$$

The general solution of the fourth-order differential equation above is  $u(z) = e^{\lambda z}$ , which substituted in Eq. (8) results in the following expression:

$$E_p I_p \lambda^4 + k_\theta \lambda^2 + k_x = 0 \tag{9}$$

The general solution constants are given by the solution of the biquadratic Eq. (9) as follows:

$$\lambda = \pm \sqrt{-\frac{1}{2} \frac{k_{\theta}}{E_p I_p}} \pm \sqrt{\frac{1}{4} \left(\frac{k_{\theta}}{E_p I_p}\right)^2 - \frac{k_x}{E_p I_p}}$$
(10)

Based on previous studies by Assimaki et al. [23] and Gerolymos and Gazetas [10–12], the distributed spring constants are proportional to the soil stiffness and foundation geometry characteristics as  $k_x \propto E_s$  and  $k_\theta \propto E_s B^2$ . As a result, for typical caisson geometries and soil–foundation impedance contrasts:

$$\left(\frac{k_{\theta}}{E_{p}I_{p}}\right)^{2} \ll \frac{k_{x}}{E_{p}I_{p}} \tag{11}$$

Based on the simplification introduced by Eq. (11) for the range of interest of the governing variables, Eq. (10) may be further simplified and the resulting solution constants may now be formulated as:

$$\lambda = \pm \sqrt{a} \pm ib = \pm (c \pm id)$$

where

$$a = -\frac{1}{2} \frac{k_{\theta}}{E_{p}I_{p}}, \quad b = \sqrt{\frac{k_{x}}{E_{p}I_{p}} - \frac{1}{4} \left(\frac{k_{\theta}}{E_{p}I_{p}}\right)^{2}}$$

$$c = \sqrt{\frac{\sqrt{(a^{2} + b^{2})} + a}{2}}, \quad d = \sqrt{\frac{\sqrt{(a^{2} + b^{2})} - a}{2}}$$
(12)

The general solution of the beam in the deformed configuration can be successively written as:

$$u(z) = A_1 e^{-cz} \cos(dz) + A_2 e^{-cz} \sin(dz) + A_3 e^{-c(D-z)} \times \cos(d(D-z)) + A_4 e^{-c(D-z)} \sin(d(D-z))$$
(13)

subjected to the following boundary conditions at the top (z = 0) given by Eq. (14), and base (z = D) given by Eq. (15) of the caisson:

$$E_p I_p u''(0) = M$$
  

$$E_p I_p u'''(0) = -(-V) = V$$
(14)

$$E_{p}I_{p}u''(D) = -(-K_{b\theta}u'(D))$$
  

$$E_{p}I_{p}u'''(D) = -(-K_{bx}u(D))$$
(15)

Note that for large D/B ratios, Eq. (13) can be simplified to approximate the response of long piles to lateral loading, by assuming that the pile does not deform below a certain critical depth. Since the displacement and rotation at the base are close to zero, the contribution of the base as part of the resistance mechanisms on the overall foundation stiffness can be neglected, and the boundary conditions in Eqs. (14) and (15) can be simplified as follows:

$$u(z=D) \to 0, \quad u'(z=D) \to 0 \tag{16}$$

Therefore, for the limit case of very long embedded foundations, the solution described by Eq. (13) is simplified to the solution for well-known governing equation for the deformation of a pile subjected to lateral loading at the top:

$$u(z) = A_1 e^{-cz} \cos(dz) + A_2 e^{-cz} \sin(dz)$$
(17)

and is subjected to the boundary conditions at the top as described by Eq. (14). As can be readily seen from Eq. (17), if large diameter caisson foundations were treated as the lower-bound limit case of piles and the distributed translational spring  $k_x$  were considered as the sole resistance mechanism at the foundation–soil interface, Eq. (17) would be further simplified to the standard equation for lateral loading of piles:

$$u(z) = \frac{e^{-\alpha z}}{2\alpha^2 E_p I_p} \left[ \left( \frac{V}{\alpha} + M \right) \cos(\alpha z) - M \sin(\alpha z) \right]$$
(18)

where

$$\alpha = \left(\frac{k_x}{4E_p I_p}\right)^{1/4}$$

Successively, employing series expansion of the products:

$$e^{-\alpha z} \cos(\alpha z) = 1 - 2\alpha z + O_2 \text{ power terms}$$

$$e^{-\alpha z} \sin(\alpha z) = \alpha z + O_2 \text{ power terms}$$
(19)

and accounting for the results of the dimensional analysis described above  $(k_x \propto E_s \text{ and } k_\theta \propto E_s B^2)$ , for high values of pile–soil impedance contrast and low values of foundation aspect ratios, i.e.,  $E_p/k_x \propto E_p/E_s \gg 1$  and low  $D/I_p^{1/4} \propto D/B$ , the maximum value that could be attained by the exponent term ( $\alpha z$ ) is:

$$\max\{\alpha z\} = \left(\frac{k_x}{4E_p I_p}\right)^{1/4} D \ll 1$$
(20)

Introducing the simplification described by Eq. (20) into Eq. (18), we can see that the higher order terms in Eq. (19) would decay much faster and hence the deformed shape could be approximated by considering terms of only first order. The resultant expression is thus similar to a rigid body response and can be expressed as:

$$u(z) = u_t - \theta z \tag{21}$$

where  $u_t$  is the translation at the top of the foundation and  $\theta$  is the rigid body rotation.

It should be noted that Eqs. (19) and (20) are here formulated to provide the limit conditions at which the response of flexible piles approaches a rigid body response for small values of  $\alpha$ , i.e., high  $E_p/E_s$  and low D/B ratios. In reality, for very low values of  $\alpha$  or low flexibility, the assumption in Eq. (16) that the base does not significantly participate in the global response is not valid, and hence a complete four-spring model with boundary conditions as described by Eqs. (14) and (15) should be used.

Successively, characteristic values are implemented in Eq. (13) (four-spring model) and Eq. (17) (two-spring model) for the distributed and concentrated springs, selected here as the static plane strain solution proposed by Novak et al. [24] for the former and the surface circular foundation stiffness on elastic halfspace proposed by Veletsos and Wei [25] and Veletsos and Verbic [26] for the latter, namely:

• Distributed translational and rotational springs (Ref. [24]):

$$\frac{k_x}{E} = 1.2, \quad \frac{k_\theta}{EB^2} = 0.3$$
 (22)

• Base translational and rotational spring (Ref. [25]):

$$K_{bx} = \frac{8GR}{2 - \nu}, \quad K_{b\theta} = \frac{8GR^3}{3(1 - \nu)}$$
 (23)

The response predicted by the four-spring and the twospring models subjected to a unit lateral load (V) at the top for three values of aspect ratio (D/B = 2, 6 and 10) is illustrated in Fig. 6a, while Fig. 6b depicts the predicted response of the model as a function of the aspect ratio (D/B) and the impedance contrast ( $E_p/E_s$ ). Based on the results shown, it can be readily seen that:

- (i) While for large aspect ratios (D/B>10), the fourspring response may be approximated by the pile solution described by Eq. (17), for D/B<10 the base has a significant effect on the response and hence cannot be neglected;
- (ii) The range of D/B ratio for which the response may be approximated by a rigid body is also a function of the impedance contrast  $E_p/E_s$ ; note that for typical reinforced concrete or steel casing and soft-medium soil deposits, this ratio is on the order of  $10^4-10^5$ ; and
- (iii) For stiffness ratios on the order of  $10^4-10^5$ , the variation in rotation along the length is less than 5% for aspect ratios D/B < 6, rendering the rigid body approximation valid.

## 5. Stiffness matrix formulation for rigid body deformed configuration

As shown above, the caisson is anticipated to respond as a rigid body for typical ranges of soil and foundation stiffness, and aspect ratios D/B < 6. Therefore, upon the application of a lateral force (V) and an overturning moment (M) at the top, the net translation and rotation



exerted by the caisson are depicted by Fig. 7, and may be adequately described in terms of the displacement at the top  $(u_t)$  and the rotation angle of the rigid pier  $(\theta)$ .

In the ensuing, the overall foundation lateral, rocking and coupled stiffness components as seen from the top of the element are formulated in a complex form, where the real term refers to the corresponding static stiffness coefficient multiplied by a factor that represents the potentially altered resistance of the foundation element when subjected to dynamic loading, and the imaginary term to the radiation of energy away from the foundation during the cyclic vibrations (radiation damping):

$$\mathscr{K}^* = \mathscr{K}_{\text{stat}}\mathscr{K}'(a_0) + ia_0\mathscr{C}(a_0) \tag{24}$$

where  $\mathscr{K}_{\text{stat}}$  is the static (push-over) stiffness  $a_0 = \omega B/V_s$  is the dimensionless frequency with  $\omega$ , B, and  $V_s$  being the loading frequency, pier diameter, and soil shear wave velocity correspondingly,  $k(a_0)$  is a frequency-dependent stiffness coefficient, and  $\mathscr{C}(a_0)$  is the frequency-dependent radiation damping component of the complex stiffness.

Under the assumption of rigid body response to transverse loading, the dynamic response at any point along the caisson is approximated by Eq. (21), namely:

$$u^{*}(z) = u_{t}^{*} - \theta^{*}z \tag{25}$$

where the superscript (\*) indicates the complex response, namely the amplitude and phase of the top displacement and rotation. Successively, requiring dynamic equilibrium of forces in the horizontal direction:

$$V^* = \int_0^D k_x^* u^*(z) \, dz + K_{bx}^* u^*(D) + \int_0^D m \ddot{u}^*(z) \, dz$$
  
=  $u_t^* [(k_x^* - \omega^2 m)D + K_{bx}^*]$   
+  $\theta^* \Big[ -(k_x^* - \omega^2 m) \frac{D^2}{2} - K_{bx}^* D \Big]$  (26)

and evaluating moment equilibrium at the top of the caisson:

$$M^{*} = -\int_{0}^{D} k_{x}^{*} u^{*}(z) z \, dz - K_{bx}^{*} u^{*}(D) D$$
  
+ 
$$\int_{0}^{D} k_{\theta}^{*} \theta^{*} \, dz + K_{b\theta}^{*} \theta^{*} - \int_{0}^{D} m \ddot{u}^{*}(z) z \, dz$$
  
$$\Rightarrow M^{*} = u_{t}^{*} \left[ -(k_{x}^{*} - \omega^{2}m) \frac{D^{2}}{2} - K_{bx}^{*} D \right]$$
  
+ 
$$\theta^{*} \left[ (k_{x}^{*} - \omega^{2}m) \frac{D^{3}}{3} + K_{bx}^{*} D^{2} + k_{\theta}^{*} D + K_{b\theta}^{*} \right]$$
(27)

Fig. 6. (a) Comparison between the predicted response of the four-spring and two-spring approximations of an embedded foundation element idealized as an Euler beam on Winkler soil subjected to a unit lateral load at the top (V = 1, M = 0), for three values of aspect ratio. (b) Normalized deformation and rotation of beam on linear elastic four-spring Winkler soil as a function of the aspect ratio (D/B), and the pier–soil stiffness contrast ( $E_p/E_s$ ), for a unit lateral load (V = 1, M = 0) applied at the top.



Fig. 7. Rigid body response (*u*) of pier upon application of lateral load (*V*) and overturning moment (*M*) at the top, described at any depth *z* from ground surface as a function of the translation at the top of the foundation  $u_t$  and the rigid body rotation  $\theta$  as  $u(z) = u_t - \theta z$ .

the following expression is formulated for the overall foundation dynamic stiffness matrix as interpreted from the top, when expressing Eqs. (26) and (27) in a matrix form:

$$\begin{bmatrix} V^* \\ M^* \end{bmatrix} = \begin{bmatrix} \mathscr{K}^*_{XX} & \mathscr{K}^*_{Xr} \\ \mathscr{K}^*_{rX} & \mathscr{K}^*_{rr} \end{bmatrix} \begin{bmatrix} u^*_t \\ \theta^* \end{bmatrix}$$
(28)

where the left-hand side of the equation represents the dynamic forcing function  $(F^*)$  applied at the top of the foundation, and the right-hand side corresponds to the product of the complex stiffness matrix  $(K^*)$  of the soil-foundation system as interpreted from the foundation top to the response vector  $(U^*)$ , namely the displacement and rotation of the caisson. In Eq. (21), the individual components of the stiffness matrix correspond to the following expressions:

$$\mathscr{K}_{xx}^{*} = (k_{x}^{*} - \omega^{2}m)D + K_{bx}^{*}$$
<sup>(29)</sup>

$$\mathscr{K}_{xr}^{*} = \mathscr{K}_{rx}^{*} = -(k_{x}^{*} - \omega^{2}m)\frac{D^{2}}{2} + K_{bx}^{*}D$$
(30)

$$\mathscr{K}_{rr}^{*} = (k_{x}^{*} - \omega^{2}m)\frac{D^{3}}{3} + K_{bx}^{*}D^{2} + k_{\theta}^{*}D + K_{b\theta}^{*}$$
(31)

Following the dimensional analysis described by Assimaki et al. [23] and Gerolymos and Gazetas [10–12], the complex stiffness terms in Eq. (28) are normalized with respect to the Young's modulus of soil ( $E_s$ ) and the diameter of pier (B), namely the stiffness and geometry characteristics of the surrounding soil and foundation element, respectively. In a dimensionless form, the equations are given as follows:

$$\begin{bmatrix} \frac{V^*}{EB^2} \\ \frac{M^*}{EB^3} \end{bmatrix} = \begin{bmatrix} \frac{\mathscr{H}^*_{xx}}{EB} & \frac{\mathscr{H}^*_{xr}}{EB^2} \\ \frac{\mathscr{H}^*_{rx}}{EB^2} & \frac{\mathscr{H}^*_{rr}}{EB^3} \end{bmatrix} \begin{bmatrix} \frac{u_t^*}{B} \\ \theta^* \end{bmatrix}$$
(32)

where

$$\left(\frac{\mathscr{K}_{xx}^{*}}{EB}\right) = \left[\left(\frac{k_{x}^{*}}{E}\right)\left(\frac{D}{B}\right) + \left(\frac{K_{bx}^{*}}{EB}\right)\right] - \left(\frac{\pi}{8(1+\nu)}\left(\frac{\rho_{c}}{\rho_{s}}\right)a_{0}^{2}\right)\left(\frac{D}{B}\right)$$
(33)

$$\begin{pmatrix} \mathscr{H}_{xr}^{*} \\ \overline{EB^{2}} \end{pmatrix} = - \left[ \frac{1}{2} \begin{pmatrix} k_{x}^{*} \\ \overline{E} \end{pmatrix} \begin{pmatrix} D \\ \overline{B} \end{pmatrix}^{2} + \begin{pmatrix} K_{bx}^{*} \\ \overline{EB} \end{pmatrix} \begin{pmatrix} D \\ \overline{B} \end{pmatrix} \right]$$
$$+ \frac{1}{2} \left( \frac{\pi}{8(1+\nu)} \begin{pmatrix} \rho_{c} \\ \rho_{s} \end{pmatrix} a_{0}^{2} \right) \left( \frac{D}{B} \right)^{2}$$
(34)

$$\begin{pmatrix} \mathscr{K}_{rr}^{*} \\ \overline{EB^{3}} \end{pmatrix} = \left[ \frac{1}{3} \left( \frac{k_{x}^{*}}{\overline{E}} \right) \left( \frac{D}{B} \right)^{3} + \left( \frac{K_{bx}^{*}}{\overline{EB}} \right) \left( \frac{D}{B} \right)^{2} + \left( \frac{k_{\theta}^{*}}{\overline{EB^{2}}} \right) \left( \frac{D}{\overline{B}} \right) \right.$$
$$\left. + \left( \frac{K_{b\theta}^{*}}{\overline{EB^{3}}} \right) \right] - \frac{1}{3} \left( \frac{\pi}{8(1+\nu)} \left( \frac{\rho_{c}}{\rho_{s}} \right) a_{0}^{2} \right) \left( \frac{D}{\overline{B}} \right)^{3}$$
(35)

and  $\rho_c$ ,  $\rho_s$  are the mass densities of the caisson foundation and surrounding soil correspondingly.

#### 6. Three-dimensional finite element simulations

The stiffness matrix described by Eq. (28) can be numerically evaluated by computing the displacements and rotations at the top of the caisson resulting from the application of a unit lateral force and a unit overturning moment, and inverting the response matrix as shown below:

$$\begin{bmatrix} u_t \\ \theta \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{xr} \\ K_{rx} & K_{rr} \end{bmatrix}^{-1} \begin{bmatrix} V \\ M \end{bmatrix} = \begin{bmatrix} u_V & u_M \\ \theta_V & \theta_M \end{bmatrix} \begin{bmatrix} V \\ M \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} K_{xx} & K_{xr} \\ K_{rx} & K_{rr} \end{bmatrix} = \begin{bmatrix} u_V & u_M \\ \theta_V & \theta_M \end{bmatrix}^{-1}$$
(36)

where  $u_V$  and  $\theta_V$  are the displacement and rotation correspondingly at the top that result from the application of a unit force, and  $u_M$  and  $\theta_M$  are the corresponding terms obtained by application of a unit moment.

This methodology, referred to as the *flexibility approach* and extensively employed for the analysis of problems in structural mechanics, is also employed in the ensuing for the calibration of the distributed and concentrated springs applicable for caisson foundations in the aspect ratio range 2 < D/B < 6 by means of 3D finite element simulations. In particular, numerical simulations are conducted in this study using the finite element software package DYNAFLOW (Ref. [27]) and validated, for the case of static loading using the software package ABAQUS (Ref. [28]). Taking advantage of the symmetry of the problem (geometry and uniaxial loading), only half of the numerical domain is simulated, thus reducing considerably the required computational effort. Further reduction in the computational effort for this configuration (namely a circular cross-section subjected to transverse loading) can be achieved by using a 2D axisymmetric model and decomposing the asymmetric loading in terms of Fourier modes with respect to the angular coordinate. Nonetheless, since such analysis is not feasible for other cross-sections such as square and rectangle, the 3D model is selected in this case to allow generalization of the macroelement development framework presented in the ensuing.

Both the soil formation and caisson foundation were simulated using 3D continuum soil elements (eight-node brick elements) in this study, while for the purpose of this work, linear elastic material models were implemented and perfect bonding was assumed at the interface, i.e., no separation was allowed under tensile stress. This assumption is equivalent to that of a cylindrical foundation 'welded' in soil, as described by Kausel [20]. The unit lateral load was applied at the center of top of caisson, whereas the moment was applied using vertical loads at diametrically opposite circumferential nodes on the top of caisson.

A direct Crout-column solver, namely a form of Gauss elimination also known as LU factorization, was implemented for the simulated configurations to minimize the effects of numerical integration instabilities in the analytical approximations to be developed on the basis of the numerical simulations. Note that the Crout-column method is based on the decomposition of the matrix of unknowns into a lower triangular matrix (L), an upper triangular matrix (U) and a permutation matrix (P), the latter of which is used in the matrix inversion in the solution of linear equation systems. The various aspects of the numerical discretization and simulation of the problem under investigation are briefly described in the following sections.

#### 6.1. Truncated numerical domain shape and discretization

Mesh sensitivity studies were conducted for the element size, far-field shape and far-field distance. For the static simulations, comparison between DYNAFLOW simulations with fixed far-field and ABAQUS simulations with infinite elements in the far-field indicated that implementation of lateral far-field distance of 5B is not sufficient for the representation of the problem. Results indicated that increasing the far-field distance to 10B provided sufficiently accurate results with less than 5% discrepancy in the estimated response by means of the two alternative solutions.

Two types of meshes were compared, which are schematically depicted in Fig. 8. In mesh-type A, the farfield is cylindrical in shape, and the computational domain discretization is finer in regions close to the pier resulting in accurately depicted stress distribution at the soil-foundation interface and coarser towards the far-field, with element size proportional to the distance from caisson axis. Also, the element thickness was selected to be constant with depth from ground surface and equal to 0.25B, while the finite element length-to-width ratio was also retained less than 3 throughout the mesh to avoid numerical distortion effects. On the other hand, in mesh-type B, the location of far-field boundaries: mesh A with cylindrical far field and MESH B with rectangular far-field.

far field is rectangular in shape, the computational domain discretization is almost uniform throughout the model, the elements are for the most part cubical in shape with dimension 0.25B, and the transformation from cylindrical to rectangular shape is performed gradually with distance from the caisson axis towards the far-field.

Benchmark static and dynamic simulations were conducted with both mesh-types, and while results were found to be in excellent agreement for adequately large truncated numerical models, the adaptive mesh-type A was selected for the purpose of this work due to the advantages associated with increased computational efficiency. In particular, the adaptive mesh discretization of model A where the finite element size increases with distance from the pier, results in a numerical model that comprises of the minimum required number of elements and correspondingly reduces the associated computational time. Nonetheless, the resulting element size that increases with distance from the pier implies that the accurate representation of wave propagation restricts the far field to a maximum distance from the foundation center, namely the distance where the element size equals the maximum element size permitted by frequency consideration (2.5B in this case). Beyond the element size constrained far-field however, numerical attenuation is observed as a result of the element size, whose contribution is beneficial by artificially increasing the effective radiation damping and

Fig. 8. Sensitivity analysis of finite element mesh discretization and



hence improving the performance of the far-field truncated conditions. A complete description of the far-field dynamic boundary conditions is provided in the ensuing.

The numerically accurate representation of wave propagation problems susceptible to numerical attenuation because of under sampling requires at least six to seven elements per wavelength. Based on this requirement,  $\lambda_{\min}/6 \ge \kappa B$  where  $\kappa$  is the ratio of size of largest element to *B*, and  $\lambda_{\min}$  is the highest frequency component of interest propagating wavelength. As a result, the maximum dimensionless frequency that can be simulated with sufficient accuracy is described by the following expression:

$$a_0 = \frac{\omega_{\max}B}{v_s} = \frac{2\pi f_{\max}B}{v_s} = \frac{2\pi B}{\lambda_{\min}} \le \frac{2\pi}{6\kappa_{\min}}$$
(37)

For the case of Model A implemented in the foregoing simulations, the ratio of the maximum element size to foundation width is  $\kappa = 0.67$ , and therefore the maximum dimensionless frequency represented is approximately  $a_{\text{max}} \approx 3.5$ . Based on the aforementioned highest accurately represented frequency, the minimum time step is consequently given by the following expression:

$$t = \frac{\kappa B}{v_s} = \frac{\kappa B}{f\lambda} \leqslant \frac{1}{6f} = \frac{T_{\min}}{6}$$
(38)

In the simulations presented in the ensuing of this study, a time step of  $T_{\rm min}/20$  to  $T_{\rm min}/40$  has been employed to ensure the accurate representation of the propagating wavelengths.

# 6.2. Far-field boundary conditions: forced vibrations and seismic incident motion

While fixed boundary were implemented for the purpose of the static analyses conducted in this study, their use was prohibited for the case of dynamic or transient response problems by the resulting spurious reflections of the elastic waves from the boundary towards the truncated numerical domain, an effect particularly pronounced in the absence of material damping. Therefore, alternative boundary conditions were investigated and their relative advantages and disadvantages are briefly described in the ensuing.

It is well known that for the case of one-dimensional (1D) simulations, effective truncation of the numerical domain may be achieved at any distance from the disturbance origin by means of viscous damping elements at the boundary. On the other hand, for 2D and 3D simulations of wave propagation in an unbounded medium, considering that the stress at any point within the domain is proportional to the particle velocity at that location as follows:

$$\sigma_i = \frac{\dot{u}_i}{\rho V} \tag{39}$$

where  $\sigma_i$  is the stress (compressional or shear),  $\dot{u}_i$  is the particle velocity at the point of interest,  $\rho$  is the density of the material of propagation, and V is the waveform

propagation velocity within the medium, implementation of dashpots with coefficients  $C_P = \rho V_P$  in the direction of the compressional wavefront propagation and  $C_S = \rho V_S$ parallel to the shear wave polarization may be used to approximate the target outgoing wave energy absorbing boundary condition in the far-field, with the exception of the following cases: (a) high angle of incidence of the outgoing waves (usually  $> 20^\circ$ ); or (b) multiple waves reaching the boundary (body, surface, etc.). For the simulations conducted in this study, both surface and body waves are generated at the caisson-soil interface during dynamic loading, and the resulting wave field has very high angle of incidence with respect to the boundary orientation at many locations. As a result, dashpot-type boundary conditions could not be implemented to successfully truncate the numerical domain.

The implementation of infinite elements in the far-field, namely elements with decaying shape functions for large distances (e.g.,  $e^{-x}$  or 1/x) was successively investigated. Infinite elements are typically used in conjunction with finite elements in boundary value problems defined in unbounded domains, or problems where the region of interest is small in size compared to the surrounding medium, and provide residual far-field stiffness for static problems and quiet boundaries for dynamic problems. Nonetheless, the formulation of these elements requires a priori approximate knowledge of the solution, in order for the shape functions to approximate as closely as possible the actual solution of the problem, as well as definition of the stress- or displacement-based boundary conditions at infinity (i.e., very large distance from the disturbance). Even further though, the use of infinite elements has been shown to also yield unsatisfactory results for large angles of incidence of waves at the boundaries, and was therefore prohibited for the 3D finite element problem under investigation. An overview for infinite elements is given by Bettes and Bettes [29].

For this purpose, a new type of boundary was implemented in this study, hereby referred to as sponge boundary and schematically depicted in Fig. 9a. In particular, the reflection of outgoing waves back into the region of interest is avoided by enclosing the region in the 'sponge layer' with progressively increasing damping coefficients. The mechanical sponge layer-soil impedance is approximately equal to unity thus minimizing the generation of reflected waves at the interface, while energy absorption is represented by means of Rayleigh damping increasing with distance to avoid any spurious reflections due to sudden change in impedance. The thickness of the sponge layer required for the absorption of the outgoing energy is estimated based on the anticipated frequency content of the excitation and the natural frequencies of the propagating medium, and a brief description of the process is given in the ensuing. The implementation of Rayleigh damping within the context of finite element numerical models for the solution of the wave equation in a viscoelastic medium, the damping matrix is assumed to be



Fig. 9. (a) Finite element mesh of the numerical model, depicting the truncated computational domain and energy absorbing (sponge) layer, (b) variation of Rayleigh damping ratio with frequency within the sponge boundary layer ( $\alpha = 20$  and  $\beta = 0.05$ ), and (c) variation of the wave amplitude reduction function with frequency  $A(\omega, x)$  within the sponge layer presented as a ratio of amplitude of ingoing and outgoing waves.

proportional to the mass and stiffness matrix as:

 $C = \alpha M + \beta K \tag{40}$ 

Successively, the modal damping ratio of the multi-degree of freedom system is calculated as:

$$D' = \frac{1}{2} \left( \alpha \omega + \frac{\beta}{\omega} \right) \tag{41}$$

where  $\alpha$  and  $\beta$  are the mass proportional and stiffness proportional damping coefficients correspondingly. For the propagation of a sinusoidal wave of unit amplitude and frequency  $\omega$  in the viscoelastic medium of the following form:

$$u(x,t) = e^{i\omega(t-x/v)}$$
(42)

The use of the viscoelastic correspondence principle results in the following expressions:

$$E^{*} = E(1 + i2D')$$
  

$$\Rightarrow V^{*} = \frac{V}{\sqrt{1 + 4D'^{2}}} \left[ \frac{1 + \sqrt{1 + 4D'^{2}}}{2} + iD' \right]$$
  

$$\Rightarrow \frac{1}{V^{*}} = \frac{1}{V} \left[ 1 - i\frac{2D'}{1 + \sqrt{1 + 4D'^{2}}} \right] = \frac{1}{V} [1 - i\alpha'] \quad (43)$$

where E is the elastic modulus corresponding to the type of propagating body wave in the medium, V is the

velocity of propagation of the wavefront, and  $\alpha' = 2D'/(1 + \sqrt{1 + 4D'^2})$ .

Substituting Eq. (41) that describes the complex modulus formulation into Eq. (42), which in turn describes the ampli-

tude of the outgoing waves in the viscoelastic medium, the latter may be expressed as:

$$u(x, t) = e^{i\omega(t-x/v^*)} = e^{-(\omega x/v)\alpha'} e^{i\omega(t-x/v)}$$
  
=  $e^{-(\omega x/v)\alpha'} u_0(x, t) = A(\omega, x)u_0(x, t)$  (44)

For the purpose of this study, imposing on Eq. (44) the requirement of minimization of the truncated numerical domain of the finite element model due to computational restrictions, as well as ensuring adequate distance of the sponge layer from the pier to allow for efficient simulation of far-field conditions, resulted in the optimal thickness of sponge layer to be selected as  $L_s = 2B$ .

The coefficients  $\alpha$  and  $\beta$  within the sponge layer, varying as a function of distance away from the pier, were selected under the following criteria: (i) to optimize the uniformity of damping ratio D' distribution over the frequency range of interest, which determines the  $\alpha/\beta$  ratio, and (ii) to ensure that the damped wave amplitude at the far-field boundary would be on the order of 5% of the undamped amplitude (i.e., at the truncated domain/sponge layer

Table 1 Mass and stiffness dependent Rayleigh damping coefficients used within the sponge boundary

	Radial soil layers of sponge boundary			
	1	2	3	4
χ	5	10	15	20
β	0.0125	0.0250	0.0375	0.05

Soil layers 1-4 correspond to cylindrical sections of the far-field at increasing radial distance from the caisson center.

interface), which determines the magnitudes of the coefficients  $\alpha$  and  $\beta$ . For the purpose of this study, the first criterion requirement yielded a ratio of  $\alpha/\beta = 400$ , and the variation of the resulting *D'* distribution with frequency is depicted in Fig. 9b. The requirement of 95% attenuation in wave amplitude within the sponge layer resulted in maximum values of the coefficients  $\alpha$  and  $\beta$  to be  $\alpha = 20$  and  $\beta = 0.05$ , which were successively distributed in four layers of radial thickness 0.5*B*; the resulting coefficients of each layer (1–4 from the interior domain towards the boundary) are shown in Table 1, while the resulting amplitude reduction function  $A(\omega, x)$  with frequency described by Eq. (42) is shown in Fig. 9c.

# 6.3. Far-field boundary conditions: kinematic soil–structure interaction

In order to evaluate the kinematic response of the foundation, namely the response of the large diameter caisson to the incidence of seismic waves, the input motion is prescribed directly to the region of interest in form of effective forcing functions at the base and lateral boundaries of the numerical domain bounded by sponge boundaries. The forcing functions for lateral boundaries are evaluated as the 1D response of the corresponding soil columns. The difference between the 1D motion and 2D response evaluated at the far-field is actually the scattered energy of the system, which propagates outwards from the irregularity and is absorbed by the artificial boundaries. The evaluation of consistent boundary conditions prescribed around the numerical domain of interest is based on the Substructure Theorem [3]. According to this theorem, the free-field vibration problem can be decomposed into substructures (the far-field and the soil-structure configuration, referred to as near-field) as shown schematically in Fig. 10.

Since the excitation is exactly the same for the far-field and the interaction problem, differences in the interface displacements  $(\Delta U = U_b - U_b^*)$  are solely attributed to differences in the interface stresses  $(\Delta S = S_b - S_b^*)$ . If the far-field is now subjected to forces  $\Delta S$ , in the absence of seismic excitation, displacements  $\Delta U$  will be produced, such that  $\Delta S = X \times \Delta U$ , where X is the frequencydependent dynamic impedance matrix of the far-field, i.e., the stiffness of the far-field as seen by the interface.



Fig. 10. Schematic representation of the Substructure Theorem for soil–structure interaction problems (the interaction problem is shown on the top, and the free-field problem on the bottom figure).

Substituting the forces and displacement differences at the boundaries, we obtain:

$$-S_b = -\mathcal{X}U_b + \mathcal{X}U_b^* - S_b^* \tag{45}$$

Since the domain is infinite, the equivalent spring stiffness implied by X is zero. The stresses  $\mathcal{X}U_b^* - S_b^*$ correspond to the far-field motion and are applied to the lateral boundaries. For the wave-propagation problem analyzed herein, the far-field motion is defined as the response of a one-dimensional soil column, subjected to the input motion prescribed at the base of the two-dimensional configuration. Successively, the fictitious forces prescribed at the lateral boundaries of the three-dimensional model are determined as follows for the case of SH-wave incidence:

- (a)  $S_b^*$  corresponds to the vertical reaction preventing the vertical motion at the far field boundary; and
- (b)  $\mathscr{X}U_b^* = \rho V_s \dot{U}_b^*$  corresponds to the product of the calculated far field (1D) response and impedance, where  $\dot{U}_b^*$  the velocity time history at the 1D column nodes, and  $V_s$  the S-wave velocity at the corresponding location.

For the purpose of this study, the forces are applied in form of surface loads (tractions) both at the base and the lateral boundaries in the 3D model. It should be noted that the Substructure Theorem is based on the principle of superposition, and is therefore applicable to linear problems as well as approximately applicable to moderately inelastic systems.

#### 6.4. Comparison with embedded foundation and pile theories

It has been shown in Eq. (30) that the global stiffness matrix is symmetric, i.e., the off-diagonal coupled stiffness

terms  $(K_{xr} = K_{rx})$  are equal, and consequently, the flexibility matrix that results from inversion of a symmetric matrix is also expected to be symmetric. Nonetheless, since 3D solid elements were used for the numerical representation of the caisson, the rotation at the top could not be measured directly by computing the gradient of lateral displacement, and was therefore calculated indirectly using the displacements of the center points at the top and bottom of caisson as  $(u_t - u_b)/D$ . The latter therefore is based on the assumption of a rigid body deformation and represents effectively the average or mean rotation of the caisson over its length.

If the responses were indeed perfectly rigid (the limit of the material stiffness to infinity), the average rotation would be the same as the rotation at the top of caisson, however, as the flexibility of caisson increases, the average rotation starts deviating from the true rotation at the top of the caisson which can also be observed in Fig. 6b. Readily stemming from this concept, while one of the cross-terms in the flexibility matrix of Eq. (36) corresponds to a rotation at unit load, the second one is obtained as a displacement at unit moment and is thus not subjected to the averaging of the response along the length of the foundation. As a result, therefore, the deviation between the two cross-terms may be used as an indication of the error introduced by the rigidity assumption of the caisson, and Fig. 11 depicts the sensitivity of the coupled stiffness  $K_{xr} = K_{rx}$  to the predicted system response, namely the variation of the expression:

$$\frac{d\mathscr{K}_{xr}}{\mathscr{K}_{xr}^{\text{avg}}} = 2\frac{\mathscr{K}_{xr} - \mathscr{K}_{rx}}{\mathscr{K}_{xr} + \mathscr{K}_{rx}}$$
(46)

as a function of the aspect ratio D/B for soil Poisson ratio v = 0.3 and foundation-soil impedance ratios  $E_p/E_s = 10^4$  and  $10^5$ , representative of the cases of concrete and steel pier, respectively. Note that for D/B ratios larger than 6, the deviation is shown to exceed 0.05 (5%) for  $E_p/E_s = 10^4$ , and as a result, the assumption of a rigid pier is considered valid only for aspect ratios  $D/B \leq 6$ .

Within the aforementioned region of applicability of the rigid response approximation in terms of material



Fig. 11. Deviation of pier from rigid behavior for different pier-soil impedance ratios  $(E_p/E_s)$ , illustrated by means of the coupled stiffness sensitivity as a function of the aspect ratio D/B for two foundation-soil impedance contrasts.

stiffness contrast and foundation geometry characteristics, Fig. 12a–c shows the comparison of results obtained for the global stiffness matrix of the caisson foundation via 3D finite elements with existing formulations by Kausel [20], Wolf [30] and Elsabee and Morray [4] developed for shallow foundations, by Davidson [31] and Gerolymos and Gazetas [10–12] for caisson foundations, and by means of a four-spring model with base springs equal to the expressions by Veletsos and Verbic [26] for a surface foundation on elastic halfspace, and shaft resistance by Novak et al. [24]. As can be readily seen, the models proposed by Kausel [20], Elsabee and Morray [4], Wolf [30], and Davidson [31] fail to capture at least one of the three global stiffness terms

![](_page_13_Figure_9.jpeg)

Fig. 12. Comparison of global stiffness matrix components of foundation as interpreted from the top of the caisson between FE simulations and embedded foundation published formulations.

for the foundation system. On the other hand, stiffness values predicted by springs obtained from combination of formulations by Novak et al. [24] and Veletsos and Verbic [26], namely constant spring values independent of the foundation aspect ratio (D/B) based on plane-strain theory, is a good approximation to the configuration investigated here. Nonetheless, while the latter combined approach captures qualitatively the overall variation of stiffness as a function of D/B, it quantitatively predicts lower stiffness values than the actual ones obtained by means of 3D finite element simulations.

Successively, Fig. 13a shows the location of center of rotation of pier as a function of eccentricity of applied loading obtained from 3D simulations and by means of the Euler beam theory (four-spring formulation). It should be also noted herein that results showed that the four-spring approximation of the pier may capture the true response much better than the two-spring model, while the observed deviation from the two-spring model was shown to increase as D/B ratio decreased and base effects would become more pronounced; the latter observation is depicted in Fig. 13b.

Based on the aforementioned conclusions, it is clear that neither the embedded foundation nor the pile existing

![](_page_14_Figure_5.jpeg)

Fig. 13. (a) Location of center of rotation from 3D FE analysis and analytical solution (four-spring beam theory) and (b) comparison of center of rotation predictions via the four-spring proposed model, and the twospring simplified representation, where the base resistance mechanism contributions are ignored.

models may be used to capture all the three global modes of soil resistance for the case of large diameter embedded (caisson) foundations. There exists therefore a clear need to calibrate the springs of the proposed model, which may be successively employed for the evaluation of the overall foundation stiffness; evaluating the lateral, rocking and coupled stiffness at the top of the foundation. The calibrated dimensionless spring constants may successively be used in analyses of structural response, replacing the continuum formulation of the infinite domain and foundation element by the foundation–soil stiffness matrix.

# 7. Calibration of Winkler spring model for caisson foundations

The distributed lateral springs along the length and concentrated spring at the base of the pier, namely  $k_x$  and  $K_{bx}$  correspondingly, were in this study estimated from the finite element-based simulations by equating the overall lateral and coupled stiffness of the pier shown in Eqs. (33) and (34) to the numerically evaluated stiffness as follows:

$$k_{x} = 2\left(\frac{\mathscr{H}_{xx}D + \mathscr{H}_{xr}}{D^{2}}\right), \quad K_{bx} = -\left(\frac{\mathscr{H}_{xx}D + 2\mathscr{H}_{xr}}{D}\right)$$
(47)

Successively, the overall rotational stiffness of the foundation as interpreted from the top, described in Eq. (35), can be expressed in the following form:

$$\mathscr{K}_{rr} - \left(k_x \frac{D^3}{3} + K_{bx} D^2\right) = k_\theta D + K_{b\theta}$$
(48)

It should also be noted that for the estimation of the distributed rotational  $(k_{\theta})$  and base rotational  $(K_{b\theta})$  spring functions, the equality described by Eq. (47) was evaluated for two different values of D within very close proximity to each other, based on the inherent stability assumption of the corresponding resistance mechanisms with (D/B), which was successively verified by means of the finite element simulations for the range of aspect ratios of interest. The resulting variation of the four spring values as a function of the foundation aspect ratio is shown in Fig. 14.

As shown in Fig. 14, resolving the spring functions by means of the flexibility approach based on the finite element-based estimated response leads to the observation that the base rotation spring  $K_{b\theta}$  contribution decreases significantly with D/B, and becomes negligible for D/B>0.75. Similar behavior is observed for the variation of the distributed rotational spring  $k_{\theta}$  for aspect ratios D/B>5-6. The trend observed is directly stemming from the procedure employed here for the derivation of the spring functions: as the aspect ratio D/B increases, the contribution of  $k_{\theta}$  and  $K_{b\theta}$  to the global rotational stiffness  $K_{rr}$  compared to  $k_x$  and  $K_{bx}$  decreases. As a result,  $K_{rr}$ 

below:

![](_page_15_Figure_2.jpeg)

![](_page_15_Figure_3.jpeg)

- (ii) The base lateral spring  $K_{bx}/EB$  evaluated by means of finite elements is found to be 0.92 for surface foundations (D/B = 0), which is in excellent agreement with the value predicted by the formulation of Veletsos and Verbic [26] for the horizontal impedance of circular foundations on elastic halfspace; for D/B > 0ratios,  $K_{bx}$  increases, a behavior attributed to the socalled trench effect according to which, the soil at deeper layers is more constrained as compared to the surface and therefore mobilizes a higher shear resistance.
- (iii) The value of the distributed rotational spring  $k_{\theta}$ increases with D/B and becomes approximately constant for 4 < D/B < 6. The increase is explained by the increase in confinement due to trench effect and higher shear resistance mobilized at the sides. The contribution of this mechanism of soil resistance significantly reduces for D/B > 6, namely in Zone III where the contribution of this mechanism to global stiffness matrix starts becoming negligible.
- (iv) The base rotational spring  $K_{b\theta}/EB^3$  has a value of 0.18 in the very low aspect ratio  $D/B \ll$  (shallow embedded foundation) region, which is in very good agreement with the theoretical rocking stiffness predicted by the formulation by Veletsos and Verbic [26] for surface foundations. Nonetheless, since the relative contribution of this spring to total rocking stiffness is very small, it may be neglected with no loss of accuracy in the solution for D/B > 1. For the case of an end bearing foundation with  $E_b$  as the base layer stiffness, as  $E_b/E_s$  contrast increases the value of lateral base resistance  $K_{bx}$  also increases in the same proportion as the base rotational spring  $K_{b\theta}$  and for very large

![](_page_15_Figure_7.jpeg)

Fig. 14. (a) Variation of distributed and concentrated spring functions with aspect ratio D/B, backcalculated from the 3D finite element simulations by equating the numerically computed and analytically derived global stiffness coefficients of the soil-foundation system as interpreted from the top. (b) Comparison of distributed lateral spring stiffness normalized by the soil's Young's modulus with existing formulations (Refs. [12,33]).

becomes increasingly insensitive to changes in values of these two springs for higher D/B ratios, a fact that renders the interpretation of their corresponding functions by means Eq. (48) (i.e., equating the analytically and numerically derived global rotational stiffness  $(K_{rr})$ to derive the variation of  $K_{b\theta}$  and  $k_{\theta}$ ) cumbersome for large values of the aspect ratio (i.e., D/B = 0.75 for  $K_{b\theta}$ , and D/B = 5-6 for  $k_{\theta}$ ). As a result, for the aspect ratio region beyond which the rocking stiffness becomes insensitive to changes in value of the aforementioned springs, the corresponding mechanism of soil resistance may be neglected altogether, which results in a simplified analytical model of the caisson foundation. Based on this interpretation, the response of pier can be broadly classified into three main zones depicted in Fig. 14a, namely Zone I for D/B = 0-2 (four-spring model), Zone II for D/B = 2-6 (three-spring model), and Zone III for D/B > 6 (two-spring model). The behavior of individual contrast ratios, the global rocking stiffness is predominantly controlled by the base resistance mechanisms represented by  $K_{bx}$  and  $K_{b\theta}$ . Nonetheless, since the relative contribution of  $K_{bx}$  (which is an increasing function of D/B ratio) to the rocking stiffness is much higher than  $K_{b\theta}$ , the error introduced by neglecting  $K_{b\theta}$ is less than 5% even for D/B as low as 2 and  $E_b/E_s$  as high as 10<sup>3</sup>.

It should be also noted that for aspect ratios D/B > 6 and relative foundation/soil stiffness corresponding to concrete piers, the response of the foundation is shown to deviate significantly from the perfectly rigid assumption. In this aspect ratio range, the caisson behaves as a flexible foundation, and the response can be estimated by means of the p-y spring approach. Note that for a linear elastic medium, the p-y curve is represented by the distributed lateral spring  $k_x$ . Using the spring values of the simplified three-mechanism analytical model obtained by means of finite element simulations, simplified expressions are derived by means of least-square curve fitting for the aspect ratio range under investigation (namely D/B = 2-6) as follows:

$$\frac{k_x}{E} = 1.828 \left(\frac{D}{B}\right)^{-0.15} \tag{49}$$

$$\frac{K_{bx}}{EB} = 0.669 + 0.129 \left(\frac{D}{B}\right) \tag{50}$$

$$\frac{k_{\theta}}{EB^2} = 1.106 + 0.227 \left(\frac{D}{B}\right) \tag{51}$$

The fitted expressions of the results are shown in Fig. 15a–c for the lateral distributed, base concentrated lateral and distributed rotational stiffness, respectively. Sensitivity analyses investigating the effects of Poisson ratio variability in the predicted response indicated that the overall stiffness terms are for the most part insensitive to changes in Poisson ratio. Therefore, the variation of spring coefficients with Poisson ratio is neglected here without loss of accuracy for the range of aspect ratio of interest, i.e., the spring functions described above are evaluated independent of the value of Poisson ratio.

# 7.1. Frequency-dependent soil resistance: calibration of dynamic springs

For the calibration of the dynamic spring functions, the flexibility approach described above was also implemented here according to which unit force and moment sinusoidal functions were applied at the top of the foundation to evaluate the dynamic response. Successively, the amplitude and phase difference of displacements and rotations were measured, and the response was expressed by means of complex functions that were next used to estimate the complex springs.

![](_page_16_Figure_10.jpeg)

Fig. 15. Least-square curve fitting for distributed translational (top), base translational (middle), and distributed rotational (bottom) spring variation as a function of D/B evaluated by means of 3D FE simulations.

In equivalence to the definition of the global stiffness matrix components shown in Eq. (24), the dynamic springs are also defined as  $K^* = K_{\text{stat}}k'(a_0) + ia_0C(a_0)$ , and the numerically estimated variation of stiffness coefficient  $k'(a_0)$  and normalized damping parameter  $C(a_0)$  as a function of the dimensionless frequency  $a_0$  estimated in this study is shown in Fig. 16a-f. The numerically derived variation is shown to fluctuate with frequency, a behavior most probably attributed to local resonances within the truncated numerical domain due to the finite dimensions of the model, as well as potential scattered energy from the absorbing far-field boundaries. Nonetheless, for the range of aspect ratios (D/B) investigated here, the variation of spring coefficients with dimensionless frequency was approximated by the expressions of Eq. (52), which based on successive comparison with numerical analyses were shown to yield satisfactory results in approximating the

![](_page_17_Figure_1.jpeg)

Fig. 16. Variation of numerically estimated values and fitted functions of dynamic stiffness and radiation damping coefficients in Eqs. (51) and (52) as a function of the dimensionless frequency  $a_0$ .

response of caisson foundations subjected to dynamic loading:

$$k'_{x} = 1 - 0.1a_{0}, \quad k'_{bx} = 1, \quad k'_{\theta} = 1 - 0.225a_{0}$$

$$\frac{C_{x}}{E} = \begin{cases} 1.85a_{0} & a_{0} < 1\\ 1.85 & a_{0} > 1\\ 1.85 & a_{0} < 1 \end{cases}$$

$$\frac{C_{bx}}{EB} = \begin{cases} 0.6a_{0} & a_{0} < 0.6\\ 0.36 & a_{0} > 0.6 \end{cases}$$
(52)

It should be also noted here that the variation of attenuation coefficients for the distributed rotational springs  $C_{\theta}$  attain negative values in the frequency range of interest (Fig. 16f), an effect indicating that the waves produced by the side shear resistance are out of phase with those produced by other resisting mechanisms. As a result, the wavefield produced by the lateral distributed shear resistance (i.e., the mechanism corresponding to the lateral rotational springs) destructively interferes with the wavefield produced by the translational mechanisms, thus obstructing the energy radiation away from the system. The distributed rotational attenuation coefficient  $C_{\theta}$ increases with frequency for values of normalized frequency, and successively decreases for  $a_0 > 1$ . Finally, for  $a_0 > 2$ , it attains values approximately zero, indicating that no energy is being radiated towards the far-field as a result of the rocking mechanism along the shaft of the caisson. The rotational attenuation coefficient is also found to increase in proportion to the aspect ratio (D/B), a behavior that may be approximated by the following expression:

$$\frac{C_{\theta}}{EB^2} = \begin{cases} -0.21 \left(\frac{D}{B}\right) a_0 & a_0 < 1\\ -0.21 \left(\frac{D}{B}\right) (2 - a_0) & 1 < a_0 < 2\\ 0 & a_0 > 2 \end{cases}$$
(53)

The validity of the approximate expressions derived above is evaluated in the ensuing by comparison of the analytically predicted response computed by means of the fitted expressions to the numerically predicted response of the foundation–soil system.

#### 7.2. Example of application: multi-layered soil deposit

An example of application of the Winkler model developed above is next illustrated, where the configuration shown in Fig. 17a is subjected to a lateral load and moment at the top, the static and dynamic response of the soil-foundation system is evaluated both by means of the three-spring Winkler model and three-dimensional finite elements, and the results are compared to illustrate the applicability of the model for the analysis of large diameter embedded foundations in multi-layered soil formations. The foundation element investigated is embedded in a three-layer linear elastic formation with Young's moduli equal to  $E_1 = 10$  MPa,  $E_2 = 30$  MPa, and  $E_3 = 50$  MPa and density  $\rho_1 = 1500$  kg/m<sup>3</sup>,  $\rho_2 = 1600$  kg/m<sup>3</sup>, and  $\rho_3 = 1800$  kg/m<sup>3</sup> for the top, middle, and bottom layers, respectively. The Poisson's ratio of all three layers is common and is equal to v = 0.3. The thickness of the top

![](_page_18_Figure_2.jpeg)

Fig. 17. (Left) Geometry and symbol definition of the static and dynamic forced excitation example of application. (Right) Comparison between peak displacement at top ( $u_i$ ) and rotation ( $\theta$ ) values evaluated via the fitted spring functions (four-spring model) and the 3D finite element simulations used for their calibration for the case of a multi-layered soil deposit.

and middle layers is  $d_1 = 3$  m and  $d_2 = 4$  m correspondingly, which overlie a linear elastic halfspace (layer 3). The diameter of caisson is B = 2 m and the depth of embedment is equal to D = 8 m.

A static lateral load of V = 1000 kN and an overturning moment of M = 2000 kN m are initially applied at the top of the caisson. Using the three-spring model developed above, Eqs. (49) and (51) yield the following values for the distributed translational and rotational springs along the shaft of the foundation for within each of the three layers (indicated by means of the subscript i=1, 2 or 3):

$$k_{xi} = 1.83 \left(\frac{D}{B}\right)^{-0.15} E_i \Rightarrow k_{x1} = 14.9 \text{ MPa}$$
  
 $k_{x2} = 44.6 \text{ MPa}, \text{ and } k_{x3} = 74.3 \text{ MPa}$ 

$$k_{\theta i} = \left[1.11 + 0.23 \left(\frac{D}{B}\right)\right] E_i B^2 \Rightarrow k_{\theta 1} = 81.2 \text{ MPa m}^2$$
$$k_{\theta 2} = 243.6 \text{ MPa m}^2, \text{ and } k_{\theta 3} = 406 \text{ MPa m}^2$$

Also, the concentrated translational spring at the base of the foundation is evaluated by means of Eq. (50) by substituting the stiffness of the third soil layer as follows:

$$K_{bx} = \left[0.67 + 0.13 \left(\frac{D}{B}\right)\right] E_3 B = 119 \text{ MPa m}$$

Using the corresponding layer thickness of the three formations in Eqs. (33)–(35), the following quantities are estimated for the overall soil–foundation system as interpreted from the ground surface:

$$\mathscr{H}_{xx} = k_{x1}D_1 + k_{x2}(D_2 - D_1) + k_{x3}(D_3 - D_2) + K_{bx}$$
  
= 416.4 MPa · m

$$\mathscr{H}_{xr} = -\left[k_{x1}\frac{D_1^2}{2} + k_{x2}\left(\frac{D_2^2 - D_1^2}{2}\right) + k_{x3}\left(\frac{D_3^2 - D_2^2}{2}\right) + K_{bx}D_3\right]$$
  
= -2468.3 MPa \cdot m<sup>2</sup>

$$\mathcal{K}_{rr} = k_{x1} \frac{D_1^3}{3} + k_{x2} \left( \frac{D_2^3 - D_1^3}{3} \right) + k_{x3} \left( \frac{D_3^3 - D_2^3}{3} \right) + K_{bx} D_3^2 + k_{\theta 1} D_1 + k_{\theta 2} (D_2 - D_1) + k_{\theta 2} (D_3 - D_2) = 18257.5 \,\mathrm{MPa} \cdot \mathrm{m}^3$$

where  $D_1 = d_1 = 3$  m,  $D_2 = d_1 + d_2 = 7$  m, and  $D_3 = d_1 + d_2 + d_3 = 8$  m. In matrix form, the displacement-force relationship of the system is therefore expressed as follows, and the resulting response expressed in terms of the top displacement  $(u_t)$  and rotation  $(\theta)$  of the foundation element are:

$$\begin{bmatrix} u_t \\ \theta \end{bmatrix} = \begin{bmatrix} \mathscr{H}_{xx} & \mathscr{H}_{xr} \\ \mathscr{H}_{rx} & \mathscr{H}_{rr} \end{bmatrix}^{-1} \begin{bmatrix} V \\ M \end{bmatrix} = \begin{bmatrix} 1.53 \text{ cm} \\ 0.00218 \text{ rad} \end{bmatrix}$$

Three-dimensional finite element simulations for the same configuration and loading functions were next conducted, and results were found to be in excellent agreement (with 10% accuracy) with the analytically estimated static response, namely:

$$\begin{bmatrix} u_t \\ \theta \end{bmatrix}_{\text{FEM}} = \begin{bmatrix} 1.78 \text{ cm} \\ 0.00239 \text{ rad} \end{bmatrix} \approx \begin{bmatrix} u_t \\ \theta \end{bmatrix}_{\text{three-spring}}_{\text{Winkler}} = \begin{bmatrix} 1.53 \text{ cm} \\ 0.00218 \text{ rad} \end{bmatrix}.$$

Successively, a dynamic sinusoidal load of amplitude V=1000 kN and frequency f=5 Hz was applied at the top of the foundation. The shear wave velocity and dimensionless frequency corresponding to the loading frequency of the transverse excitation for each layer is

estimated as follows:

$$V_{si} = \sqrt{\frac{E_i}{2(1+v)\rho_i}} \Rightarrow V_{s1} = 50 \text{ m/s}, \quad V_{s2} = 85 \text{ m/s}$$
  
and  $V_{s3} = 103 \text{ m/s},$ 

$$a_{0i} = \frac{2\pi f B}{V_{si}} \Rightarrow a_{01} = 1.25, \quad a_{02} = 0.74, \text{ and } a_{03} = 0.61.$$

Using Eqs. (52) and (53), the dynamic distributed translational and rotational springs and concentrated lateral spring at the base are computed as follows:

$$k_{x1}^* = 13.04 + 18.5i, \quad k_{x2}^* = 41.30 + 41.07i, \text{ and} \\ k_{x3}^* = 69.77 + 56.42i$$

 $K_{hx}^* = 119.0 + 36.0i$ 

$$k_{\theta_1}^* = 58.36 - 25.2i, \quad k_{\theta_2}^* = 203.04 - 74.59i, \text{ and} \\ k_{\theta_3}^* = 350.27 - 102.48i$$

and the resulting global stiffness matrix as interpreted from the top of the foundation is computed using the complex formulation of Eqs. (33)–(35) as:

$$\begin{bmatrix} \mathscr{H}_{XX}^* & \mathscr{H}_{Xr}^* \\ \mathscr{H}_{rX}^* & \mathscr{H}_{rr}^* \end{bmatrix} = \begin{bmatrix} 393.1 + 312.0i & -2359.9 - 1615.8i \\ -2359.9 - 1615.8i & 17351.3 + 9498.4i \end{bmatrix}$$

The dynamic displacement–load relation yields the following results for the maximum displacement and rotation of the caisson foundation, which are below compared to the corresponding values estimated by means of finite element dynamic simulations:

$$\begin{bmatrix} u_t^{\max} \\ \theta^{\max} \end{bmatrix}_{\text{three-spring}} = \begin{bmatrix} 1.12 \text{ cm} \\ 0.00162 \text{ rad} \end{bmatrix} \approx \begin{bmatrix} u_t^{\max} \\ \theta^{\max} \end{bmatrix}_{\text{FEM}}$$
$$= \begin{bmatrix} 1.4 \text{ cm} \\ 0.00186 \text{ rad} \end{bmatrix}$$

Fig. 17b and c shows the comparison between maximum displacement and rotation values computed using the fitted springs as described above, and the response evaluated directly by means of 3D finite element simulations for a frequency range of 1-10 Hz. As can be readily seen, the assumption of individually responding soil layers and resulting approximation of independently responding Winkler elements along the shaft of the model may be employed with no significant loss of accuracy for the analysis of large diameter embedded foundations in multilayered soil profiles, which represents one of the basic assumptions of this study. In particular for the dynamic case, despite the fact that the presence of multiple layers causes reflections of waves at the interfaces arising from the impedance contrast, the radiated waves are shown to propagate for the most part parallel to the layer interfaces (except for the base spring for which they are normal to the interface). As a result, conditioned on the approximation that the soil profile layers act almost as waveguides for the energy emanating away from the pier, namely on a finite impedance contrast between the layers of the idealized profile, the anticipated reflections and their effect on the total response is minimized, rendering the assumption of this study valid.

# 8. Kinematic soil-structure interaction of large diameter embedded foundations

The inability of a stiff foundation to comply with the deformation field imposed by the soil response to seismic incident waves and the resulting motion incompatibility between the foundation response and the free-field is translated into forces and moments applied on the foundation. In turn, the rigidly responding foundation causes filtering of the far-field motion, a phenomenon that materializes for wavelengths comparable or shorter than the dimensions of the foundation. This effect referred to as kinematic interaction and schematically depicted in Fig. 18 for the case of large diameter embedded foundation investigated here, may be expressed in terms of the following transfer functions that illustrate the foundation response resulting from the imposed free field motion (i.e., the soil column deformation in absence of the foundation):

$$H_u(a_0) = \frac{u_t}{u_{ff}} \tag{54}$$

$$H_{\theta}(a_0) = \frac{\theta B}{u_{ff}} \tag{55}$$

In order to evaluate the analytical expressions for the transfer functions describing the foundation response subjected to seismic incident waves, consider a sinusoidal

![](_page_19_Figure_19.jpeg)

Fig. 18. Schematic illustration of kinematic soil–structure interaction due to motion incompatibility between the far-field and the rigid foundation.

vertically propagating anti-plane shear wave (SH) incident from the underlying halfspace. The solution of the far-field displacement field in this case is given by:

$$u_{ff}(z) = u_{ff} \cos\left(2\pi \frac{z}{\lambda}\right) = u_{ff} \cos\left(a_0 \frac{z}{B}\right)$$
(56)

where z is the depth from the surface,  $\lambda$  is the wavelength of the propagating sinusoid, and  $a_0$  is the corresponding dimensionless frequency. For the configuration shown in Fig. 15, equilibrium requirement of forces in the horizontal direction results in the following expression:

$$u_{t}^{*}((k_{x}^{*} - m\omega^{2})D + K_{bx}^{*}) + \theta^{*}\left(-(k_{x}^{*} - m\omega^{2})\frac{D^{2}}{2} - K_{bx}^{*}D\right)$$
$$= u_{ff}^{*}\left(k_{x}^{*}\frac{B}{a_{0}}\sin\left(a_{0}\frac{D}{B}\right) + K_{bx}^{*}\cos\left(a_{0}\frac{D}{B}\right)\right)$$
(57)

Similarly, requiring moment equilibrium at the top of the foundation results in the following expression:

$$u_t^* \left( -(k_x^* - m\omega^2) \frac{D^2}{2} - K_{bx}^* D \right)$$
  
+  $\theta^* \left( (k_x^* - m\omega^2) \frac{D^3}{3} + K_{bx}^* D^2 + k_{\theta}^* D + K_{b\theta}^* \right)$   
=  $u_{ff}^* \left[ k_x^* \left( \frac{B}{a_0} \right)^2 \left( \left( 1 - \cos\left(a_0 \frac{D}{B}\right) \right) - a_0 \frac{D}{B} \sin\left(a_0 \frac{D}{B}\right) \right)$   
-  $K_{bx}^* D \cos\left(a_0 \frac{D}{B}\right) - k_{\theta}^* \left( 1 - \cos\left(a_0 \frac{D}{B}\right) \right)$   
-  $K_{b\theta}^* \frac{a_0}{B} \sin\left(a_0 \frac{D}{B}\right) \right]$  (58)

Eqs. (56) and (57) may be successively expressed in a matrix form as follows:

$$\begin{bmatrix} \frac{\mathscr{K}_{xx}^{*}}{EB} & \frac{\mathscr{K}_{xy}^{*}}{EB^{2}} \\ \frac{\mathscr{K}_{xy}^{*}}{EB^{2}} & \frac{\mathscr{K}_{rr}^{*}}{EB^{3}} \end{bmatrix} \begin{bmatrix} \frac{u_{l}^{*}}{B} \\ \theta^{*} \end{bmatrix} = \begin{bmatrix} \frac{V_{\text{eff}}^{*}}{EB^{2}} \\ \frac{M_{\text{eff}}^{*}}{EB^{3}} \end{bmatrix}$$
(59)

where the effective force vector corresponds to the following quantities:

$$\frac{V_{\text{eff}}^*}{EB^2} = \frac{u_{ff}^*}{B} \left( \frac{k_x^*}{E} \frac{1}{a_0} \sin\left(a_0 \frac{D}{B}\right) + \frac{k_{bx}^*}{EB} \cos\left(a_0 \frac{D}{B}\right) \right), \tag{60}$$

$$\frac{M_{\text{eff}}^*}{EB^3} = \frac{u_{ff}^*}{B} \left[ \frac{k_x^*}{E} \left( \frac{1}{a_0} \right)^2 \left( \left( 1 - \cos\left(a_0 \frac{D}{B}\right) \right) - a_0 \frac{D}{B} \sin\left(a_0 \frac{D}{B}\right) - \frac{K_{bx}^*}{EB} \frac{D}{B} \cos\left(a_0 \frac{D}{B}\right) - \frac{k_{\theta}^*}{EB^2} \right) \right]$$
$$\times \left( 1 - \cos\left(a_0 \frac{D}{B}\right) - \frac{K_{b\theta}^*}{EB^3} a_0 \sin\left(a_0 \frac{D}{B}\right) \right]$$

By definition of the transfer functions given by Eqs. (54) and (55), the following expression is derived for the estimation of the foundation response upon the incidence of a sinusoidal (monochromatic) wave resulting in free-field

response of amplitude  $u_{ff}$ :

$$\begin{bmatrix} H_u(a_0) \\ H_\theta(a_0) \end{bmatrix} = \begin{bmatrix} \left| \frac{u_t^*}{u_{ff}} \right| \\ \left| \frac{\theta^* B}{u_{ff}} \right| \end{bmatrix} = \left| \begin{bmatrix} \frac{\mathscr{K}_{xx}^*}{EB} & \frac{\mathscr{K}_{xr}^*}{EB^2} \\ \frac{\mathscr{K}_{xr}^*}{EB^2} & \frac{\mathscr{K}_{rr}^*}{EB^3} \end{bmatrix}^{-1} \begin{bmatrix} \frac{V_{\text{eff}}^*}{EBu_{ff}^*} \\ \frac{M_{\text{eff}}^*}{EB^2 u_{ff}^*} \end{bmatrix} \right|$$
(61)

Three-dimensional finite element numerical simulations were here conducted to evaluate the target transfer functions  $H_u$  and  $H_\theta$  for the case of vertically propagating SH waves. For the numerical representation of the problem, the far-field motion was applied in by means of effective forcing functions in the interior of the truncated computational domain, while the potentially scattered wavefield was verified to be almost entirely absorbed by the sponge boundaries as described in the foregoing sections. A comparison with the values derived by means of the analytical expression using both fitted and numerically derived (interpolated) spring constants is shown in Fig. 19a and b for D/B = 4.

Results obtained in this section illustrate that the analytical solution for the kinematic response of rigid intermediate embedded foundations obtained by means of the three-spring model, allows the kinematic interaction effects of pier foundation elements to be captured within an acceptable degree of accuracy, considering the substantial reduction in computational time. In particular, while the 3D finite element simulations do not capture quantitatively the amplitude of the rocking response, the model does predict the resonant and destructive interference frequencies (i.e., the maxima and minima) of the corresponding transfer function within acceptable range of accuracy. It

![](_page_20_Figure_17.jpeg)

Fig. 19. Displacement  $(H_u)$  and rotational  $(H_\theta)$  transfer functions for kinematic SSI of a weightless foundation with aspect ratio (D/B=4).

should be also noted at this point that the values predicted by three-spring Winkler model are for the most part conservative, namely the predictions represent less pronounced reduction in the translational motion and higher induced rocking motions. Finally, the analytical transfer functions obtained by means of the model described above as well as the results of 3D finite element simulations are shown to be in the bounded range predicted by means of the shallow foundation theory (Ref. [4]) and the pile kinematic response as formulated by Gazetas et al. [32]: this result implies that the proposed model is indeed an improvement to the existing models employed for the simulation of rigid deep foundations subjected to seismic motion, and by means of a procedure that substantially reduces the computational effort, the model can be used to approximately predict the foundation response subjected to transient seismic excitation given the aspect ratio of the foundation D/B.

### 8.1. Transient foundation response to non-stationary seismic incident motion

The analytical expressions developed above for the displacement and rotation transfer functions from the far-field deformation caused by vertically propagating SH seismic waves to the rigid pier translation and rotation may be implemented at minimal computational cost in a computer script and used to estimate the motions corresponding to any transient loading by means of Fourier reconstruction of the response signal, provided that the medium of propagation is linear elastic or moderately nonlinear. Fig. 20a and b shows an example of seismic displacement time history prescribed at the freefield base for a caisson (B = 2 m, D = 8 m) embedded in homogenous halfspace (Young's modulus, E = 1.0 MPa, Poisson's ratio, v = 0.3, and mass density,  $\rho = 1500 \text{ kg/m}^3$ ) and the corresponding Fourier transform. For the numerical simulations, the far-field (1D) response is initially computed by means of a 1D soil column model, and the resulting time histories are imposed at the corresponding location of the numerical domain (i.e., at the soil-sponge boundary interface) in the form of effective forces. Results are compared in the ensuing to the corresponding ones obtained by means of Fourier analysis using the transfer functions described in Eqs. (60) and (61) and the dynamic fitted springs developed in the foregoing sections.

In particular, comparison between the analytically predicted and numerically evaluated response is depicted in Fig. 21a and b. Note that the numerically obtained time histories have been shifted in time to account for the wave propagation duration of the excitation traveling from the far-field to the pier. Results presented above readily illustrate that that the model is able to capture the response of pier within an acceptable degree of accuracy while substantially reducing the computational time and numerical modeling aspects of the problem that may strongly affect the computed response such as the selection of

![](_page_21_Figure_5.jpeg)

Fig. 20. Displacement time history (top) and corresponding Fourier spectrum (bottom) prescribed at the base of the numerical domain for the illustration of the model performance in predicting the foundation response to transient excitation.

proper integration scheme and time-step, finite element model discretization and effective absorbing boundary conditions. Nonetheless, while the dominant frequency in both translational and rocking motion response evaluated by means of the analytical model is found to be in excellent agreement with the numerical results, the maximum translation predicted by the analytical model is 0.37 m (which is higher compared to the numerically obtained value  $u_{\text{max}} = 0.23$  m), and the pier rotation is predicted to be 0.34 rad by means of the analytical model (higher than the corresponding value of 0.22 rad obtained by means of the numerical simulations). It should be noted though that while incompatibilities between analytical and numerical results stem from the overprediction of the system stiffness by means of the four mechanisms of resistance that do not capture the complete physical problem, the approximate model is expected to yield conservative results for the predicted translations and rotations compared to 3D finite element simulations and does provide a improved approximation to the existing simplified models available for the evaluation of kinematic response of intermediate rigid embedded foundations.

### 9. Conclusions

We have described the development of an analytical model for the prediction of the response of rigid cylindrical caisson foundations characterized by aspect ratios D/B = 2-6 and embedded in linear elastic soil media, using a simple Winkler spring model with four springs. Results illustrated in this paper show that for the range of aspect ratio of interest (D/B = 2-6), the effect of base rotational

![](_page_22_Figure_2.jpeg)

Fig. 21. Comparison of translation (top) and rotation (bottom) of caisson foundation when subjected to transient motion, evaluated via the analytical three-spring model and the 3D finite element simulations in amplitude variation with time (left) and frequency content (right).

spring is indeed negligible and therefore, a simplified threespring model may be used instead to capture the pier response. Based on this approximation, expressions were developed for the three springs as functions of aspect ratio (D/B) and the dimensionless loading frequency since sensitivity analyses showed that the effects of Poisson ratio may be neglected in the evaluation of the aforementioned springs. Results of the static response of a configuration embedded in multi-layered soil obtained by means of the proposed model were compared to 3D finite element simulations, and the analytically evaluated response was found to be in excellent agreement with the numerically derived values at a considerably reduced computational effort.

Analytical expressions were developed for the global foundation stiffness matrix and for the transfer functions of kinematic interaction effects, for the prediction of forced vibration and seismic transient response correspondingly. The theoretical values of free-field/pier response transfer functions for translational and rocking motion resulting from the seismic excitation were compared to the values obtained by means of 3D finite element simulations, and despite the fact that the proposed formulation does not simulate the pier response exactly attributed to the complex load transfer mechanisms applied at the soil–foundation interface that cannot be captured by the simplified threespring proposed model—it may be applied to capture the important response parameters, namely the frequency content and evolution of time-history variation.

Finally, it should be noted that while the Winkler spring functions presented above are applicable for linear elastic medium with no material damping, the effect of material damping can be accounted for by means of the elasticity–visco-elasticity correspondence principle, namely the use of complex soil moduli of the form  $E^* = E(1+2i\xi)$ , where  $\xi$  is

the material damping ratio. Furthermore, while the formulation presented in this paper is not capable of capturing the separation between the soil and pier interface, modified Winkler springs that include a stiffness element, a damper and a contact element with Coulomb friction and low tension resistance may be implemented to account for the separation at the soil-foundation interface, and the variability in soil strength subjected to compression vs. extension. The approach may be even further extended to capture the nonlinear soil behavior by implementing nonlinear spring elements that represent the macroscopic response of the material as a function of depth. It should be noted also that in particular for the representation of kinematic interaction, a first approximation to the nonlinear response of the soil-foundation system (provided that the foundation material is responding within the linear elastic range) could be obtained by means of equivalent linear analyses in the far-field, successively to be used as effective function at the base and soil-foundation interface for the reduced stiffness and material damping evaluated at convergence of the algorithm. Based on the developed computational platform, analytical formulations may also be developed for the motion-response transfer functions of the pier subjected to horizontally propagating coherent SH waves or for the response of intermediate rigid foundations of various cross-sectional geometries.

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